Mechanical Characteristics of Rock Mass under Loading for Retrofit of Underground Fallout Shelters for Protection of Civil Population

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Abstract

Nowadays there's still the questions in context of global strategy about prevention and protection of civilian population and critical infrastructure even when in the present doctrine the global powers "forgets" about the old strategy of nuclear warfare. That means that we still should think about retrofit those of underground spaces that was decommissioned in last years and to start thinking about reuse of mining shafts in rock mass as a fallout shelters for protection of civil population. By rock mass we mean the volume of rock necessary to represent a model of the structure of the rocks enclosed in it and determined by a specific task related to the impact of external forces (boundary conditions) on this volume. Currently, there is no universally accepted point of view on the possibility of an approach to the processes of massifs deformation; rocks being under load in various geological conditions from a single common point, the question now arises of creating the most realistic mechanical model of the massif. In this paper were summarizing some mathematical apparatus for rocks mechanical characteristics within the massif loading processes for underground structures model.

KEY WORDS: rock properties, mathematical models, compressive strength, fallout

1. Introduction

Use of mechanical characteristics determined for individual rock samples cannot give a practical answer for the numerical solution of problems in which the individual areas of a rock mass have to be considered. It is generally accepted that the main difference between rock properties in samples and in the mass is caused by the presence of various types of fracturing. In this case, corrections for the effect of fracturing are introduced to the calculation method using the parameters of a solid body.

Essentially, the main difference in the properties of an individual sample and the properties of the massif is a qualitative change in the structure of the deformable elements of the massif caused by the presence of fracturing and separates when moving a certain part of the massif as affected by external forces during mining, i.e., with one or another type of loading of the rocks mass. It is necessary to distinguish between the concepts: properties of rocks in the massif and properties of the massif of rocks.In the first case, we are talking about changes in the properties of rocks, which are usually determined for samples, when they are examined in a mass condition, which causes first of all, a change in the stress and strain state of the rock. Characteristics of the rocks properties in this case are similar to the characteristics determined on the samples (compressive strength, tensile strength, proportional modulus, Poisson's ratio, etc.).

The properties of the massif primarily reflect the general picture of the massif, its structure and can have parameters completely different from the generally accepted ones [1]. Obviously, the processes occurring in the massif under the effect of external forces determine both the mechanical model of the massif and its properties. The mechanical model of the massif, thus, determines the method for solving the problem set. Based on the foregoing, it is advisable to introduce a more accurate idea of the rock mass [2].

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2. Methodology

Consider a planar mechanical symbolic diagram of a certain state of a roof massif after development without fixing. Five zones with qualitatively different characteristics of a loaded rock mass can be distinguished in the diagram. Since each of these zones differs from the others in their qualitative structural characteristics, it is necessary in the calculations to apply the theory for the corresponding zone that reflects its model with sufficient approximation [3]. Naturally there is no clear distinction of zones in the massif as its structure gradually changes from zone to zone and there are areas with mixed properties.

The parameters of the properties of the rock mass will be determined by the selected mechanical model of the massif structure. Each new quality that changes the conditions of massif deformation requires changes in the theoretical descriptions of the mechanical model and determines the composition of the characteristics of the massif. The primary reason for the collapse of roof rocks after cutting lava can be considered the excess of tensile stresses in the lower layers of the roof tensile strength. We find the conditional stress distribution in the roof mass using the solution for an elliptical hole, the edge of which is subject to uniform pressure [4].

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The solution is considered in the polar coordinates ρ and θ on the plane ζ, which is a conformal transformation of the plane.

The solution to the problem is represented as

$$\rho\rho = -P + \frac{P(\rho^3-1)^3(\rho^2+1)}{(\rho^4-2\rho^2\cos2\theta+1)^2};$$

$$\theta\theta = -P + \frac{P(\rho^4-1)(1+2\rho^2+\rho^4-4\rho^2\cos2\theta)}{(\rho^4-2\rho^2\cos2\theta+1)^2}.$$

(1)

If we take the rectangular coordinates of OXY so that the Ox axis coincides with the ρ axis, and the OX axis coincides with the θ axis, then

$$\rho\rho = \sigma'_y; \qquad \theta\theta = \sigma'_x.$$

(2)

Then for $\cos2\theta = -1$

$$\sigma'_y = -P + \frac{P(\rho^2-1)^3}{(\rho^2+1)^3}; \quad \sigma'_x = -P + \frac{P(\rho^2-1)(\rho^4+6\rho^2+1)}{(\rho^2+1)^3}.$$

(3)

In our case, when the roof contour is free of load, this solution can be applied if we add stress expressions (3) that reduce pressure on the edge of the roof, i.e.

$$\sigma_y = \sigma'_y + \gamma H; \quad \sigma_x = \sigma'_x + \lambda\gamma H.$$

(4)

where λ is the coefficient of lateral thrust; γ is the specific gravity of the rocks; H is the depth of development. Then, to express normal stresses in the roof mass, we will have

$$\sigma_y = \frac{\gamma H(\rho^2-1)^3}{(\rho^2+1)^3}; \quad \sigma_x = -\gamma H(\lambda-1) + \gamma H\frac{(\rho^2+1)(\rho^4+6\rho^2+1)}{(\rho^2+1)^3}.$$

(5)

Note that, with a lateral thrust coefficient $\lambda = 1$, the value of normal stresses σx will be

$$\sigma_x = \frac{(\rho^2+1)(\rho^4+6\rho^2+1)}{(\rho^2+1)^3}\gamma H.$$

(6)

Use of equations (5) allows you to calculate the zone of rocks in the roof, subject to tensile stresses.

3. Time Dependent Formulation of the Solid Phase

The formulation of the structure of lining involves time dependent change of temperature and stress states in each lamina, which simulate approximations of temperature, internal pressure, inertia forces, etc. The structure is loaded by external and internal pressures and change of temperature and the outer boundary is emerged into the Winkler medium or the surrounding rock is simulated by a particular layer. Axisymmetric initial strains or eigenstrains (simulating temperature and pressure) are uniform in each layer. These may be caused, for example, by thermal changes, inelastic deformation of the concrete matrix during curing or under volume weight load. The purpose of the analysis is to find simple expressions for the average stresses caused by the mechanical loads and by the eigenstrains in the layers. A cylindrical $r\theta z$ system of coordinates is defined according to Figure 1. The inner and outer boundaries of the structure are denoted as Γ_a at $r = a$ and Γ_b at $r = b$, respectively, where a < b, and the end faces as Γ_L at $z = L$ where $z = 0$ at the origin of the coordinate system. Similarly, for each layer (j), the inner and outer radii are denoted as $r = a_j$ and $r = b_j$, where $a_j < b_j$. It follows that $b_j = a_{j+1}$ for j = 1,2,…,N - 1, and that $a_j = a$ and $b_j = b$.
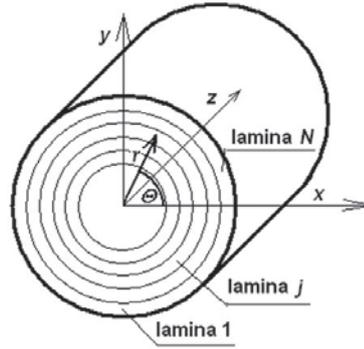

Fig.1. Geometry of laminated composite cylinder

Each layer is assumed to be at most cylindrically orthotropic. Specific elastic constants for the layers can be derived from those of plane layers or sublaminates, with appropriate coordinate transformations, [6]. Since the relations in each lamina can be asymmetric, the constitutive laws for the material in each layer can be written as,

$$\boldsymbol{\varepsilon}^j = \begin{pmatrix} \varepsilon_{rr}^j \\ \varepsilon_{\theta\theta}^j \\ \varepsilon_{zz}^j \end{pmatrix} = M^j \begin{pmatrix} \sigma_{rr}^j \\ \sigma_{\theta\theta}^j \\ \sigma_{zz}^j \end{pmatrix} + \begin{pmatrix} \mu_{rr}^j \\ \mu_{\theta\theta}^j \\ \mu_{zz}^j \end{pmatrix} = M^j \boldsymbol{\sigma}^j + \boldsymbol{\mu}^j \tag{7}$$

where the strain tensor $\boldsymbol{\varepsilon}^j$ is recorded as a vector, similar to the stress tensor $\boldsymbol{\sigma}^j$ and the uniform eigenstrains $\boldsymbol{\mu}^j$ in layer j. Recall that the eigenstrains can describe not only the change of temperature, but also irreversible processes in the solid phase, such as plasticity, visco-plasticity, prestress, etc. Compliance matrix M^j of the cylindrically orthotropic material is written in terms of the elastic constants defined in the r-coordinates, as

$$M^j = \begin{bmatrix} M_{rr}^j & M_{r\theta}^j & M_{rz}^j \\ M_{r\theta}^j & M_{\theta\theta}^j & M_{\theta z}^j \\ M_{rz}^j & M_{\theta z}^j & M_{zz}^j \end{bmatrix} = \begin{bmatrix} 1/E_r^j & -v_{r\theta}/E_\theta^j & -v_{rz}/E_z^j \\ -v_{\theta r}/E_r^j & 1/E_\theta^j & -v_{\theta z}/E_z^j \\ -v_{zr}/E_r^j & -v_{z\theta}/E_\theta^j & 1/E_z^j \end{bmatrix} \tag{8}$$

where $v_{r\theta}^j E_r^j = v_{\theta r}^j E_\theta^j$, etc., so that $M_{pq}^j = M_{qp}^j, p,q = r,\theta,z$.

The reciprocal of equation (2) are derived as

$$\boldsymbol{\sigma}^j = \begin{pmatrix} \sigma_{rr}^j \\ \sigma_{\theta\theta}^j \\ \sigma_{zz}^j \end{pmatrix} = L^j \left[\begin{pmatrix} \varepsilon_{rr}^j \\ \varepsilon_{\theta\theta}^j \\ \varepsilon_{zz}^j \end{pmatrix} - \begin{pmatrix} \mu_{rr}^j \\ \mu_{\theta\theta}^j \\ \mu_{zz}^j \end{pmatrix} \right] = L^j [\boldsymbol{\varepsilon}^j - \boldsymbol{\mu}^j] \tag{9}$$

with $L^j M^j = I$, a (3 x 3) identity matrix.

3.1. Relations in a Single Layer

Next, let a single cylindrically orthotropic layer (j) be separated from the structure, constrained by prescribed uniform surface displacements. The constraint with the neighboring layers is given by compatibility conditions, i.e. by continuity of the inner surface displacements u_a^i and by the outer surface displacements u_b^j:

$$u_r^j(a_j) = u_a^j, \quad u_r^j(b_j) = u_b^j \tag{10}$$

Recall that the kinematical equations in cylindrical coordinates in each layer are

$$\varepsilon_{rr} = \frac{\partial u_r^j}{\partial r}, \quad \varepsilon_{\theta\theta}^j = \frac{u_r^j}{r}, \quad \varepsilon_{zz}^j = u_z^j / z = \text{const.}, \quad \varepsilon_{r\theta} = \varepsilon_{rz} = \varepsilon_{\theta z} = 0 \tag{11}$$

and that the stresses follow from equation (9) and must satisfy the equations of equilibrium in cylindrical coordinates, which reduce here to the single equation

$$\frac{\partial \sigma_{rr}^j}{\partial r} + \frac{\sigma_{rr}^j - \sigma_{\theta\theta}^j}{r} = P^j(r,t), \qquad P^j(r,t) = \rho \frac{\partial^2 u_r^j}{\partial t^2} \tag{12}$$

where ρ is the density and t is time, force P^j is the inertia force working inside of the j-lamina. The other equations are satisfied identically.

Substituting equations (9) and (11) into equation (12), we find that

$$\frac{\partial^2 u_r^j}{\partial r^2} + \frac{1}{r}\frac{\partial u_r^j}{\partial r} - k_j^2 \frac{u_r^j}{r^2} = \frac{1}{rL_{rr}^j}[(L_{rr}^j - L_{r\theta}^j)\mu_{rr}^j +$$

$$+ (L_{r\theta}^j - L_{\theta\theta}^j)\mu_{\theta\theta}^j + (L_{rz}^j - L_{\theta z}^j)(\mu_{zz}^j - \varepsilon_{zz}^j)] + P^j \tag{13}$$

where $k_j^2 = \dfrac{L_{\theta\theta}^j}{L_{rr}^j}$. In the particular case of a cylindrically orthotropic layer one finds that

$$k_j^2 = E_\theta^j(1 - v_{rz}^j v_{zr}^j)/[E_r^j(1 - v_{\theta z}^j v_{z\theta}^j)].$$

Note that the heat causing the change of temperature is very high and the multiplication by stiffness terms can be prevailing on the right side. If P is omitted, the general solution of equation (13) is

$$u_r^j(\xi_j) = (A_j \xi_j^{k_j} + B_j \xi_j^{-k_j} + \xi_j S_j)b_j \tag{14}$$

where $\xi^j = r/b_j$. Recall that b_j is the outer radius of the layer (j), and the last term is a (constant) particular integral which depends on the eigenstrains and the constant axial strain ε_{zz}^j,

$$S_j = \frac{1}{(1 - k_j^2)L_{rr}^j}[(L_{rr}^j - L_{r\theta}^j)\mu_{rr}^j + (L_{r\theta}^j - L_{\theta\theta}^j)\mu_{\theta\theta}^j + (L_{rz}^j - L_{\theta z}^j)(\mu_{zz}^j - \varepsilon_{zz}^j)] =$$

$$= S_j^1 \mu_{rr}^j + S_j^2 \mu_{\theta\theta}^j + S_j^3(\varepsilon_{zz}^j - \mu_{zz}^j)$$

providing that $k_j \neq 1$, while for $k_j = 1$ it holds

$$S_j = \log(b_j \xi_j - \frac{1}{2})[U_j(\mu_{rr}^j - \mu_{\theta\theta}^j) + V_j(\varepsilon_{zz}^j - \mu_{zz}^j)]$$

In the above formulas it should be inserted:

$$S_1^j = \frac{L_{rr}^j - L_{r\theta}^j}{L_{rr}^j(1 - k_j^2)} = \frac{L_{rr}^j - L_{r\theta}^j}{L_{rr}^j - L_{\theta\theta}^j}, \quad S_2^j = \frac{L_{r\theta}^j - L_{\theta\theta}^j}{L_{rr}^j(1 - k_j^2)} = \frac{L_{r\theta}^j - L_{\theta\theta}^j}{L_{rr}^j - L_{\theta\theta}^j},$$

$$S_3^j = \frac{L_{\theta z}^j - L_{rz}^j}{L_{rr}^j(1 - k_j^2)} = \frac{L_{\theta z}^j - L_{rz}^j}{L_{rr}^j - L_{\theta\theta}^j},$$

and

$$U_j = \frac{L^j_{rr} - L^j_{r\theta}}{2I^j_{rr}}, \quad V_j = \frac{L^j_{\theta z} - L^j_{rz}}{2I^j_{rr}}$$

It is worth noting that the constant S_j expresses the influence of eigenstrains and the free parameter ε^j_{zz} under condition that $a_j = b_j = u^j_z = 0$.

The integration constants are found from the boundary conditions (10) as

$$(1 - c_j^{2k_j})A_j = -\frac{u^j_a}{a_j}c_j^{k_j+1} + \frac{u^j_b}{b_j} + S_j(c_j^{k_j+1} - 1)$$

$$(1 - c_j^{2k_j})B_j = \frac{u^j_a}{a_j}c_j^{k_j+1} - \frac{u^j_b}{b_j}c_j^{2k_j} + S_j(c_j^{2k_j} - c_j^{k_j+1})$$

(15)

where $c_j = a_j / b_j$.

This opens the way for evaluating the layer strains and stresses from equation (9). Of interest later are the interfacial tractions. In each layer the stresses can be considered as the Lebesque average $<.>$ over the lamina (j).

If the process of loading the solid phase (tunnel lining, for example) is divided into time intervals, P^j can be created from a difference scheme for time increments and an average of displacements in each lamina j. Then the particular integral of (13) is calculated as,

$$S_j = \frac{P^j}{4 - k^2}$$

(16)

This opens the way for evaluating the layer strains and stresses from equation (9). Of interest later are the interfacial tractions. In each layer the stresses can be considered as the Lebesque average $<.>$ over the lamina (j) providing that the number N is large enough. Hence the averaged stresses can formally be recorded as,

$$\begin{pmatrix} <\sigma^j_{rr}> \\ <\sigma^j_{\theta\theta}> \\ <\sigma^j_{zz}> \end{pmatrix} = s^j \begin{pmatrix} u^j_a \\ u^j_b \\ u^j_z \end{pmatrix} + w^j \begin{pmatrix} \mu^j_{rr} \\ \mu^j_{\theta\theta} \\ \mu^j_{zz} \end{pmatrix} \qquad \begin{pmatrix} P^j_a \\ P^j_b \\ P^j_z \end{pmatrix} = k^j \begin{pmatrix} u^j_a \\ u^j_b \\ u^j_z \end{pmatrix} + q^j \begin{pmatrix} \mu^j_{rr} \\ \mu^j_{\theta\theta} \\ \mu^j_{zz} \end{pmatrix}$$

(17)

where $P^j_a = \sigma^j_{rr}(a_j)$, $P^j_b = \sigma^j_{rr}(b_j)$, $P^j_z = <\sigma^j_{zz}>V_j$, V_j is the area of the lamina (j), s^j, w^j, k^j and q^j are best evaluated numerically or using the approach described in [4].

Also one can define eigenstress λ^j as the average stresses caused in a fully constrained layer, $u^j = 0$, by the uniform eigenstrains μ^j , $\lambda^j = w^j \mu^j$. It is not necessarily identical with the definition $\lambda = -L\mu$, usually adopted in the Cartesian system, where the stresses caused by uniform eigenstrains in fully constrained homogeneous volume are also known.

It remains to involve the inertia forces P^j into the formulation. Since the complex system describing the problem is very complicated, the time steps start with lumped mass densities at the middle of laminas (if they are small enough the approximation is reasonable). To do this, "system of springs" is created and a dynamical system is defined inside of the solid phase. First, the spring stiffnesses can be derived from the model completed in the next section. The inertia force (because of the symmetry only one is to be considered) is done from the previous time step, and this force is derived as that acting to the solid as well as to the gaseous phase.

3.2. Overall Response of the Solid Structure

The laminated structure should obey continuity conditions as well as the continuity of interfacial tractions, both along the boundaries between adjacent layers. The tractions on the respective surfaces at $r = a$, $r = b$, and at $z = L$, are defined in analogy with the previous definitions of the interfacial tractions as,

$$P_a = P_a^1 = -(p_a^1 a), \ P_b = P_b^N = -(p_b^N b), \ P_z = \sum_{i=1}^{N} P_z^i = <\sigma_{zz}^i > V_i \qquad (18)$$

where the last equality reflects the force equilibrium in axial direction. Generally, two types of eigenstrains are admitted in each layer, fixed eigenstrais $\bar{\mu}^j$ and variable eigenatrains μ^j that can be adjusted according to the current state of temperature, pore pressure, and other non-forced phenomena (prestrain is a typical one). The continuity conditions of displacements and tractions lead to a classical overall relation:

$$\boldsymbol{Ku} = \boldsymbol{P}, \ \boldsymbol{u} = (u_1, u_2, ..., u_N, u_{N+1}, u_z)^\mathrm{T}, \quad \boldsymbol{P} = (P_1, P_2, ..., P_N, P_{N+1}, P_z)^\mathrm{T} \qquad (19)$$

where K is neither symmetric nor banded, T denotes transposition. On the other hand, writing the "stiffness matrix" K as

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{11}(N+1) \times (N+1) & \boldsymbol{K}_{12}(N+1 \times 1) \\ \boldsymbol{K}_{21}(1 \times N+1) & K_{22}(1 \times 1) \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_z \end{Bmatrix} \qquad (20)$$

then matrix K_{11} is symmetric, positive definite and banded tridiagonal. Vector P covers the influences of external forces and eigen parameters. The generalized plain strain can be involved into computation by applying a uniform force in z-direction and fulfill the equilibrium in the same direction.

4. Gaseous Phase

In this chapter the nonlinear equations for the gas medium (air) are briefly summed up. First, general formulation is preferred in the rectangular Cartesian coordinates to be in compliance with the standard theory by Landau, Lipshitz, [8]. Then the system is transformed to cylindrical coordinates and the time and space dependent solution is suggested. The problem defined in gaseous medium links chemistry, shock waves (gas dynamics) and temperature, [9].

4.1. Basic Equations for Gas Medium

Mathematical modeling of the air movements is based on the solution of equations of gas dynamics, which for general three-dimensional problem in Cartesian system of coordinates are listed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = M \qquad (21)$$

$$\frac{\partial(\rho c_a)}{\partial t} + \frac{\partial(\rho v_x c_a)}{\partial x} + \frac{\partial(\rho v_y c_a)}{\partial y} + \frac{\partial(\rho v_z c_a)}{\partial z} = R_\alpha + M_\alpha \qquad (22)$$

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(p + \rho v_x^2)}{\partial x} + \frac{\partial(\rho v_x v_y)}{\partial y} + \frac{\partial(\rho v_x v_z)}{\partial z} = F_x \qquad (23)$$

$$\frac{\partial(\rho v_y)}{\partial t} + \frac{\partial(\rho v_x v_y)}{\partial x} + \frac{\partial(p + \rho v_y^2)}{\partial y} + \frac{\partial(\rho v_y v_z)}{\partial z} = F_y \qquad (24)$$

$$\frac{\partial(\rho v_z)}{\partial t} + \frac{\partial(\rho v_x v_z)}{\partial x} + \frac{\partial(\rho v_y v_z)}{\partial y} + \frac{\partial(p + \rho v_z^2)}{\partial z} = F_z \qquad (25)$$

$$\frac{\partial e}{\partial t} + \frac{\partial[(e+p)v_x]}{\partial x} + \frac{\partial[(e+p)v_y]}{\partial y} + \frac{\partial[(e+p)v_z]}{\partial z} = H \qquad (26)$$

where:

x, y, z	Cartesian coordinates [m]
$V_x, V_y, V_z,$	components of the vectors of velocity, [m/msec]
ρ	density of gas [kg/m³]

P pressure of gas [MPa]

$e = \rho[\varepsilon + (v_x^2 + v_y^2 + v_z^2)/2]$ full energy of a unit of mass of the gas, [MPa]

ε potential energy [m² / ms²/]

$\dfrac{1}{2}\rho(v_x^2 + v_y^2 + v_z^2)$ kinetic energy [m² / ms²/]

R_a an instantaneous rate of time and position signifies the fact that chemical species can be created or consumed by chemical reactions (natural events).

M, F, H, M_a signify the existence of resources of mass, momentum, and energy, as well as of individual chemical species, the latter literally in addition to the naturally-occurring effects of homogeneous chemical reaction R_a .

C_a coefficient, its values are dependent on chemical potentials, [10].

If the formulation possesses cylindrical symmetry three dimensional equations (21)-(26) can be transformed into one dimensional equations of the form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial r} + \frac{\rho v}{r} = M, \qquad \frac{\partial(\rho c_a)}{\partial t} + \frac{\partial(\rho v c_a)}{\partial r} + \frac{\rho v c_a}{r} = \dot{M}_a,$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(p + \rho v^2)}{\partial r} + \frac{\rho v^2}{r} = F, \qquad \frac{\partial e}{\partial t} + \frac{\partial[(e+p)v]}{\partial r} + \frac{(e+p)v}{r} = H$$

$$\text{(27)}$$

The governing equation for the change of temperature *T* is written as:

$$C\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)$$

$$- C_w\rho_w\left(V_x\frac{\partial T}{\partial x} + V_y\frac{\partial T}{\partial y} + V_z\frac{\partial T}{\partial z}\right)$$

$$\text{(28)}$$

where

$$V_x = -K_x\frac{\partial p}{\partial x}, \quad V_y = -K_y\frac{\partial p}{\partial y}, \quad V_z = -K_z\frac{\partial p}{\partial z}$$

$$\text{(29)}$$

$$= K_{ox}\exp(-\eta\sigma_{xe}), \quad K_y = K_{oy}\exp(-\eta\sigma_{ye}), \quad K_z = K_{oz}\exp(-\eta\sigma_{ze})$$

$$\text{(30)}$$

with *T, C,* and λ are the temperature, the volumetric heat capacity and the thermal conductivity of the appropriate medium, respectively. $K_{0x}, K_{0y}, K_{0z}, \eta$ are constants received from laboratory tests.

Equation describing the explosion can be recorded as

$$p = (\gamma - 1)\rho\varepsilon, \qquad \text{(31)}$$

where γ is the exponent of adiabatic process. For the air γ = 1.4, in case of explosion the exponent of adiabatic process becomes density dependent, i.e. γ = γ(ρ). Exponent of adiabatic process γ(ρ) can be calculated in the following way

- γ = 3 if ρ > 440 kg/m³;
- γ = 1.3 if ρ < 50 kg/m³ ;
- γ = γ(ρ) if $50 \leq \rho \leq 440$ kg/m³ - linear interpolation can be applied (monotonic and smooth dependence on density ρ is assumed).

$$\log\left(\frac{T}{T_0}\right) = d_1 + md_6 + (d_2 + md_7)Y +$$
$$+ (d_3 + md_8)Z + (d_4 + md_9)YZ + (d_5 + md_{10})Z^2 \tag{32}$$

where $m = \{1 + \exp[d_{11}(Z + d_{12})]\}^{-1}$, $T_0 = 288.16_{0K}$, $X = \log(p/101.3)$ $Y = \log(\rho/1.225)$, the pressure is given in

kPa, $Z = X - Y$ and the coefficients $d_1, d_2, ..., d_{12}$ are found in the paper, [11].

According to the same paper γ in relation (24) can be expressed as:

$$\gamma = a_1 + na_9 + (a_2 + na_{10})Y + (a_3 + na_{11})Z +$$
$$+ (a_4 + na_{12})YZ + a_5Y^2 + a_6Z^2 + a_7YZ^2 + a_8Z^3 \tag{33}$$

where $n = \{1 + \exp[(a_{13} + a_{14}Y)(Z + a_{15}Y + a_{16})]\}^{-1}$, $Y = \log(\rho/1.292)$, and $Z = \log(\varepsilon/78408.4)$.

The unit for ε is m2/s2 and the coefficients $a_1, a_2, ..., a_{16}$ are given in [12, 13]. Here the original relation (31) is applied.

5. Conclusions

In this paper were summarizing some mathematical apparatus for rocks mechanical characteristics within the massif loading processes for underground structures model.

Mathematical modeling of the air movements was based on the solution of equations of gas dynamics, which was presented for the general three-dimensional problem in the Cartesian system of coordinates.

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