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Theory of Chaos in Transport

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Abstract

The paper deals with the theoretical problems of nonlinear dynamical systems with internal parameters, which can be called chaotic systems with some simplification. The science of chaos has a relatively short history and is not yet sufficiently reflected by experts in various fields, resp. The authors are confused with the terms confusion, coincidence, etc. The authors believe that in examining chaos, it is necessary to consistently distinguish systems, namely deterministic, stochastic, and chaotic. Specifically, possible and anticipated practical applications of chaos theory with a focus on the issue of congestion of traffic flows in road transport.

KEY WORDS: *outcome, top event, gates, cut set, risk analysis, application*

1. Introduction

Historically, chaos theory can be dated to the early 20th century, in the studies of the French mathematician, physicist, and philosopher of science Henri Poincaré on the problem of motion of three objects with mutual gravitational force, the so-called problem of three bodies. Poincaré discovered that there may be orbits that are non-periodic, which are neither constantly increasing nor approaching a fixed point. The problems of three bodies, turbulence in gases and liquids, nonperiodic oscillations in radio circuits, etc. have no theoretical support to explain their non-standard behavior. Chaos theory was advancing rapidly after the middle of the twentieth century, when it became clear to some scientists that linear theory, the prevailing theory of systems in this period, simply could not explain the observed behavior in certain experiments. An electronic computer became the main catalyst for the development of chaos theory. Most mathematical theories of chaos involve simple iterations, the calculation of which is practically impossible without the use of computers. When modeling the weather forecast, the American meteorologist and mathematician Edward Lorenz came to the surprising result that in the subsequent repeated model, the forecast weather was completely different than in the original model. In examining why this is the case, a very simple cause was discovered [1]. The original variable model was rounded to 3 decimal places, but the computer worked with 5 decimal places. This difference is very small and should not have a practical effect on the solution. However, Lorenz discovered that small changes in initial conditions lead to dramatically large changes in output.

The common meaning of chaos is disorder, instability, randomness, unpredictability, imbalance, behavior without obvious law, etc. In the exact sciences, chaos theory deals with the behavior of nonlinear dynamical systems, which under certain conditions may exhibit a phenomenon known as deterministic chaos, extremely sensitive to initial conditions. Due to the sensitivity, the behavior of these chaos-like systems appears to be random, even though they are deterministic systems. In general, the principle is that if the internal behavior of the general system (black box) is not known, it is not possible to determine what its essence is. However, these systems can be described exactly and mathematically defined in them. Tectonics of the earth's plates, the behavior of transport systems, economic or population development, etc. Systems showing deterministic chaos are usually very complex. The meaning of the word chaos in the field of natural exact sciences is in great disagreement with the lay but in everyday life the usual understanding of the word chaos as a total disorder.

A nonlinear dynamic system can generally exhibit one of the following types of behavior, and that systems can always be at rest, always expand (unbounded systems), can perform periodic motion, quasi-periodic motion, chaotic motion, resp. periodic chaotic movement. The appropriate type of behavior depends on the initial state of the system and the values of its parameters.

In practice, this means that a small change in the initial conditions leads to a massively completely different

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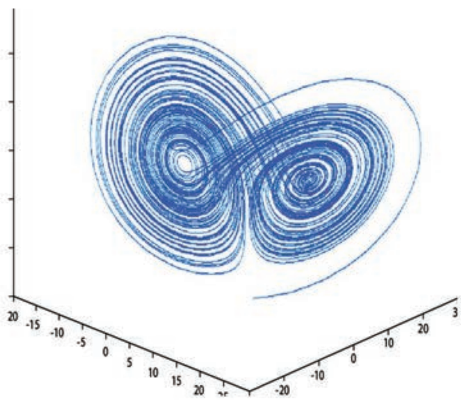
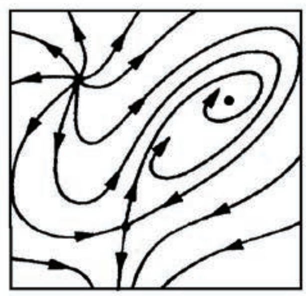
result over time. An example of such sensitivity is the well-known "butterfly effect", where the flutter of butterfly wings causes imperceptible changes in the atmosphere, which over time can lead to such dramatic changes as the occurrence of a tornado on the other side of the globe. The flutter of the butterfly's wings, as an analogy, means that a completely negligible change in the initial conditions of the system will cause a chain of events leading to large-scale to catastrophic phenomena. Sensitivity to initial conditions can be quantified over time, or Lyapunov exponent, which expresses the degree of divergence (divergence) of close trajectories in the state space of a dynamical system. Extremely sensitive dependence means that close points separate and converge infinitely, which is characteristic of chaotic dynamical systems.

An important term in chaos theory is the attractor (point or trajectory), which is the final state of the system to which the system points in time. It is one of the ways to visualize chaotic movement, resp. any type of motion, where a phase diagram of the motion is created. In this diagram, time is implicit but contained, and each axis represents one dimension of the state. If we draw the position of the pendulum with respect to its speed, when the pendulum at rest is shown as a point and the pendulum in periodic motion will be shown as a simple closed curve (orbit). Attractors (points) are often associated with dissipative systems where some element consumes energy, but this only applies to simple physical systems. However, they usually form more complex loops depending on the number of degrees of freedom. Some form fractals, so-called "strange attractors" (e.g. Fig. 1) [2], which are complex geometric objects having completely characteristic shapes (motifs), but which are generated by repeated use of simple rules. Systems with strange attractors exhibit chaotic behavior. One of the best known and most complex diagrams of chaotic systems is the Lorenz attractor (e.g. Fig. 2) [2]. Selected phase portraits of nonlinear systems are shown in Fig. 3 and Fig. 4 [3]. The phase portrait shows (trajectory) the solution of the initial value problem of a nonlinear system expressed in mathematical form, usually by an autonomous system of differential equations.

		
Fig. 1 Fractal example	Fig. 2 Lorenz attractor	Fig. 3 Possible phase portrait nonlinear system

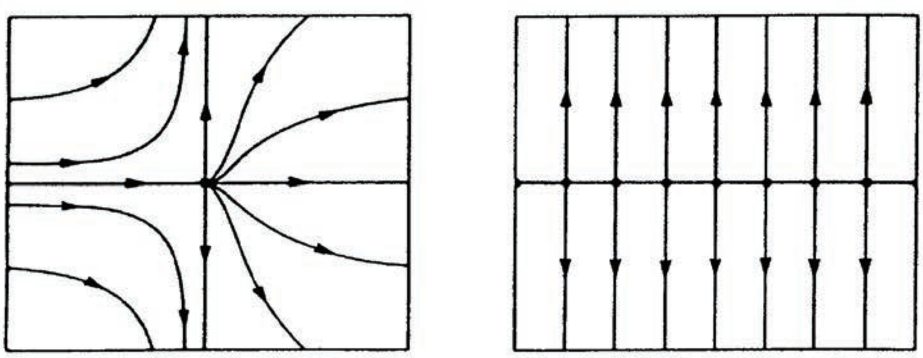


Fig. 4 Possible comparison of phase portraits of a nonlinear system and its linearization

Very interesting shapes are created in it, which can look like butterfly wings. Strange attractors occur in continuous dynamic systems if they have three or more dimensions, and also in some discrete systems, for which the condition of the number of dimensions does not apply. Complex chaotic systems are very sensitive to small changes that can disrupt the system and deviate it from equilibrium.

The dynamics of market systems can be described as two basic feedback loops and causal loops that affect various aspects of, for example, the stock market. The positive feedback loop strengthens itself, and this effect of one variable increases the other variable, which also increases the first variable.

The result can be the exponential growth of the system, its deviation from equilibrium, and the collapse of the system (bubble), which has been repeatedly confirmed in practice in history, respectively. Today, analysts expect this possibility, for example, to be in connection with inflation in the US Fed, which is expected to spread massively within a globalized world. Conversely, a negative feedback loop has a similar effect, and the system responds to a change in the opposite direction. Periods with high uncertainty do not have to be caused only by the dynamics of the system. Environmental factors, such as natural disasters, earthquakes or floods, depletion of strategic raw materials, fossil fuels, etc., can also cause market volatility, as well as personal motivation of traders (his doubts, desires, hopes, intuitions), as well as political, economic (inflation, overvalued stock prices not corresponding to GDP growth, record margins accelerating the downturn, generally overheating of markets, etc.) and the social environment.

2. Application of Chaos Theory in Road Transport

Chaos theory can find, somewhat surprisingly, application in such a purely technical and deterministic field as transport, resp. so far road transport and its very problematic part, namely traffic flows. Road transport is the efficient movement of means of transport in a transport network (a set of sections and nodes), the product of which is transport. In doing so, it uses transport technologies, which usually consist of means of transport (vehicles), transport infrastructure (transport and communication system, energy, etc.) and transport organization. The traffic flow can be understood as a purposeful relocation of a sequence of mobile means of transport, or pedestrians, etc. in the transport network. The traffic flow can be considered with some simplification as a physical system, for the description of which it is necessary to derive equations that determine the models of its behavior. This can be done in various ways, from the classical Newtonian theory of gas flow, resp. Fluids as a continuum, with consideration of force action and interaction of relations between them with the help of vector algebra through Hamiltonian approach and use of Hamiltonian, e.g., with canonical transformations and the Hamilton-Jacobi equation, or Lagrange formalism. However, examples of individual options are beyond the scope of this paper. In general, traffic flow models have evolved from a purely Newtonian search for independent relationships between traffic flow parameters (speed, intensity, density), through the introduction of current dynamics by time description of parameters to the use of gas kinetics as a basis for mathematical modeling and computer verification of models on the issue of congestion. When modeling with fluid flow, standard physical flow equations with appropriate boundary conditions are used, networks for discretization of the current field using algebraic equations with interpolation functions, resp. partial differential equations. This is usually followed by a specific numerical solution, using several methods, i.e. finite element methods, finite volumes, finite differences, boundary elements, etc. An important characteristic of a section of the transport network is throughput, which is, for example, the maximum number of vehicles that can leave an intersection in a given direction per time unit and continue to move to the next section of the networks. Current traffic flow models use a macroscopic approach (Traffic Stream Model) with categories of variables: density, intensity, and medium speed, a microscopic approach with categories distance between vehicles, speed of individual vehicles and possibly sub models, e.g., for behavior, to predict traffic flows in the road transport network. drivers and/or vehicles with categories: free driving, car-following, lane changing, various macroscopic and microscopic models (linear, logarithmic, exponential, mode, two-mode, diffuse, etc.). These models are physically usually based on theories of gas and fluid flow, which allows to use unambiguous relationships between traffic intensity and traffic speed movement, or traffic intensity and traffic flow. The movement of the vehicle in the column for the dependence speed, time and the movement space is shown in Fig. 5 [1], [4]. The relationship between the intensity and density of the traffic flow is shown in Fig. 6 [1], [4]. The relationship between speed and traffic flow density is shown in Fig. 7 [1], [4]. The relationship between the intensity and speed of the traffic flow is shown in Fig. 8 [1], [4]. The focus of applications for chaos theory in the area of traffic flows is primarily congestion (congestion, congestion) in the road network. There are several reasons, such as inadequacy (exceptional traffic density associated with globalization, the number of vehicles has multiplied in recent years, the number and capacity of roads is essentially the same as in 1990) or infrastructure overcrowding (congestion, motorways, border crossings, car parks, etc.), traffic accidents, vehicle failures, signaling equipment failures, shock waves, road maintenance and repairs, traffic congestion, inappropriate lane connections, emergencies, etc. When using models and vehicle movement simulations for modeling traffic congestion, when i.e., equations describing the motion of a gas or liquid with a constricted neck have repeatedly obtained "strange" results. This can be explained by the fact that vehicles do not move like gas or liquid, drivers try to avoid a traffic jam or collision by slowing down if they approach another vehicle, while gas or liquid does not behave this way. After modifying the equations used in this model, the relative accuracy, reproducibility, and agreement of the results with practical measurements on communication were confirmed. When the gas enters the constriction, the molecules accumulate, and there is compression, which then propagates backward like a shock wave. We can compare this situation to the slowing and accumulation of vehicles in queues in front of the narrowing of the traffic road. Vehicles that slow down in a narrowing will also slow down the vehicles behind them, which will cause waves of alternating stopping and starting against the direction of the traffic flow, which leads to a jump transition from smooth traffic to a traffic jam. An interesting experience that has emerged in the calculations is the fact that traffic congestion can arise spontaneously under certain conditions, without traffic congestion or other external causes, even with an increase in the number of traffic flows on the road. An example is traffic that moves freely, its density is not maximum, but suddenly switches to a synchronized traffic flow. Under certain conditions, a relatively short and small fluctuation in vehicle speed or topology can cause a traffic collapse, which can last long after the initial impulse of the traffic jam has passed. Mathematically, this means that at a certain combination of density and traffic flow, a phase break occurs, which in nonlinear dynamical systems is called hysteresis. It has been experimentally proven [5-11] that road transport fulfills the laws of chaos theory. However, this applies in particular to nonstationary traffic situations, which can be characterized by feedback processes, nonlinear dynamics, high sensitivity to small changes in initial conditions, self-organization during congestion, fractal similarity of traffic curves, similarities of traffic situations, etc.

Comment:

Self-similarity is a property of an object or system that causes the object or system, at whatever scale, to still have the same shape or different characteristics. The dimension is in this case defined as , where N is the number of copies of the original shape at L-fold magnification of the linear dimension, eg the cube has dimension 3 ($8=2^3$).

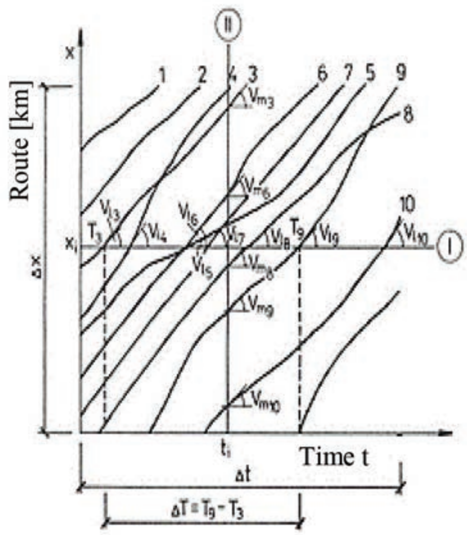


Fig. 5 Vehicle movement in a traffic convoy

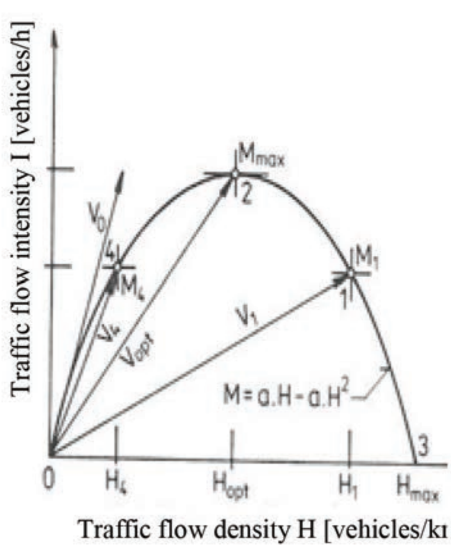


Fig. 6 Relationship between intensity and density traffic flow

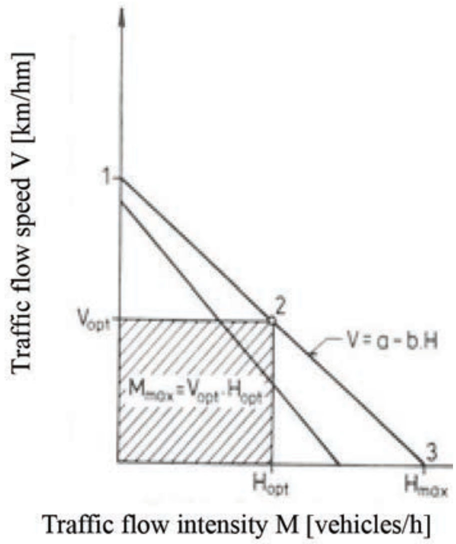


Fig. 7 Relationship between speed and density traffic flow

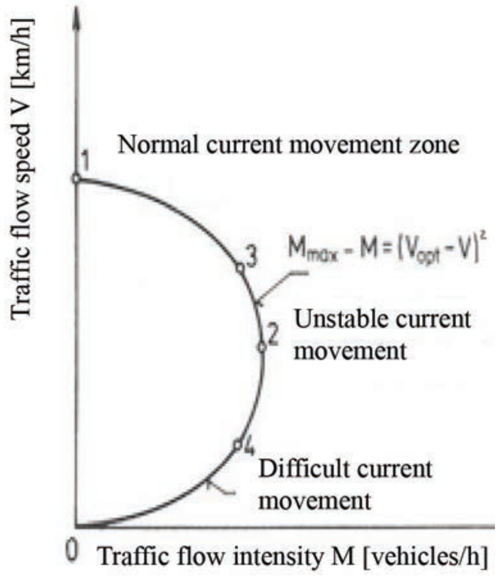


Fig. 8 Relationship between intensity and speed of traffic flow

It turns out that a collapse may occur in response to small random fluctuations in traffic flow, the real situation is probably different than projected by traffic engineers. Their ideas about the maximum capacity of motorways and 1st class roads, even at a density significantly lower than the one that means the fulfillment of the designed capacity, can lead to spontaneous traffic congestion. The current solution is to limit the entry of vehicles on crowded roads. A possible solution is probably the exact timing of the moment of entry, synchronized with the moment of decrease in vehicle density, and to compensate for the fluctuation that would lead to a phase break. An alternative solution could be autonomous vehicles, which are currently being developed very quickly and are likely to be brought back into normal operation within a few years once the legislation is completed.

Autonomous vehicles will be able to automatically control speed and spacing directly on road and highway networks and in conjunction with a central computer and sensors will automatically control the engine and brake control unit and optimize the movement of each vehicle, respectively traffic flows. The issue is very complex, because the parameters and properties that characterize the huge number of vehicles of different types, sizes, tonnages, etc., moving on the road combines a large number of common symptoms and interactions with the behavior of other objects, such as information that moves on other networks, e.g. 5G standard telecommunications mobile network (transmission speed up to 20 Gbit/s), NGA (Network Grapfic Annunciator) electronic communications network, transport telematics, Internet of Things, virtual and augmented reality, 360^o videos, holographic telephones, etc. Research [1] confirmed that there is a certain road transport paradox that the construction and addition of a new road to the existing road network may even reduce its capacity as a whole in certain circumstances.

Discussions and conclusion

Chaos theory is a complicated and, in some respects, controversial mathematical theory that seeks to explain the effect of seemingly insignificant factors. It is a relatively new scientific discipline that views the reality of the world and its laws in a completely different way from traditional sciences. For various reasons, it seems to be changing so far appreciated, because its name already arouses unwanted associations. To understand the essence of chaos theory, it is necessary to have sound knowledge of various exact disciplines, especially mathematics, physics, computer science and experimental work. Applications in transport and traffic flows seem to make sense, especially in combination with traditional methods and models. In cases of standard (ordered) traffic flow behavior, there may be phase space regions that exhibit fractal set complexity and separate boundary region regions, which is a clear feature of the presence of a nonlinear component of dynamical systems [12-16]. The tool for examining such systems is the projection of phase space into sets of lower dimensions. Phase space allows you to turn the numbers of computer simulations into images, selects a realistic dynamic system whose mechanical, especially gaseous parts move, essential information and creates maps in which the paths to the emergence of possible entities are marked. In the case of traffic flows, there is an interaction of spatial, velocity and time trajectories when they are projected into the plane, the determinism of the development of the system is lost. At the same time, in the stage of chaotic development, projection into the plane means the loss of the possibility of predicting future behavior.

Calculations and experiments are always burdened with errors, their source is different, but usually they are errors of the model and the relevant calculation procedure itself, with the fact that it is difficult to determine their share in the inaccuracies of the results. A way to determine the accuracy of the results seems to be to compare the numerical values with the results of the experiments. Analogous errors in computer modeling and simulation have historically occurred, for example, in the results of CFD (Computational Fluid Dynamic) models. Recently, it has become a standard for the use of a systems approach to the assessment of models, simulations, iterations, calculation and experiment results, which includes the possibilities of uncertainty management, credibility measurements, assessment validation, code verification) using experimental results (bench-mark) and built database. On the other hand, it was found experimentally that the system may not behave chaotically even in the case of nonlinearity, but the probability of chaotic development increases with the size of the nonlinear term of the dynamic system. For nonlinear systems that are not excited by the input signal, equilibrium steady states or periodic steady states may occur. An important feature of the nonlinear component of the system is its fractal character. Fractal geometry, unlike Euclidean geometry, is the geometry of a humanly imperfect perception of the reality of the world. Fractal geometry expresses the fact that natural objects are independent of the scale of observation in terms of shape observation, morphology, etc. From this point of view, chaos theory may have a philosophical aspect of not the world, because it shows that even very simple systems can exhibit infinitely complex behavior in terms of knowledge.

Despite the indicated difficulties, the theory of dynamic systems and the theory of chaos can be a challenge for road transport and transport telematics for solving very difficult situations, such as traffic congestion, etc., through computer simulations and subsequent experiments. Chaos theory is undoubtedly a scientific discipline, as it fulfills the reasons whose fulfillment is necessary for this statement. It is mainly the fact that chaos theory is theoretically and experimentally justified, is characterized by a specific subject and methods, techniques and procedures of research, terminology and has a clear relationship to other scientific disciplines with precise relationships expressed in mathematical, physical and other concepts or functions, etc. The existence of a developing field of study in selected technical, economic, but also humanities universities resp. research areas.

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