Journal of Security and Sustainability Issues *www.lka.lt/index.php/lt/217049/*

ISSN 2029-7017/ISSN 2029-7025 online
2013 Volume 2(4): 65–79
http://dx.doi.org/10.9770/jssi.2013.2.4(7)

OPTION PRICING USING MONTE CARLO SIMULATION

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Received 11 September 2012; accepted 16 March 2013

Abstract. Special features that options include are the main reason of their growing amounts trading in the financial markets. Options can be used in many imaginative ways to create various attractive investment opportunities. Empirical researches all over the world illustrated that options incorporate an insurance element not available in any other security and because of that they can be used by investors to create return distributions unobtainable with the strategy of allocating funds between fixed income securities and stock portfolios. But investor must understand that one of the main aspects of profitable trading in derivative securities is their proper evaluation and pricing. As the exact valuation of options is quite difficult, the article deals with the theoretical and practical aspects of pricing of options. The purpose of the research is to adopt Monte Carlo simulation method to predict prices of plain vanilla options and to compare them to real option prices and option prices calculated using analytical Black-Scholes formula.

Keywords: option contract, price, stock price, call, put, Monte Carlo simulation, Black-Scholes model.

Reference to this paper should be made as follows: Martinkutė- Kaulienė, R.; Stankevičienė, J.; Žinytė, S. 2013. Option pricing using Monte Carlo simulation, *Journal of Security and Sustainability Issues* 2(4): 65–79. http://dx.doi.org/10.9770/jssi.2013.2.4(7)

JEL Classification: G17.

1. Introduction

Though the history of trading in option contracts is quite old for the first time exchange listed options were traded in 1973. Since then, the volumes of their trade had risen sharply all over the world. This growing was determined by the special features that options include. Options can be used in many imaginative ways to create various attractive investment opportunities. Empirical researches presented in financial literature (Hull 2008, Friedentag 2000, Martin 2001, Evrim-Mandaci *et al.* 2013) illustrated that options incorporate an insurance element not available in any other security and because of that they can be used by investors to create return distributions unobtainable with the strategy of allocating funds between a stock portfolios and fixed income securities. Options can be used to speculate for profit, earn income to enhance investment returns, protect against a temporary decline in the value of a stock or other commodity both financial and material.

The pricing of option contracts is a very important area of research. Many problems in mathematical finance involve the computation of a particular integral. The primary methods for pricing options are binomial trees and other lattice methods, such as trinomial trees, and finite difference methods to solve the associated boundary value partial differential equations. So in many cases those integrals can be valued analytically, and in still more cases they can be valued using numerical integration. For example, the Black-Scholes model provides explicit closed form solutions for the values of certain (European style) call and put options.

However, when the number of dimensions in the problem is large, analytical models and numerical integrals become unavailable, the formulas exhibiting them are complicated, entail many restrictive assumptions and difficult to evaluate accurately by conventional methods. In these cases, simulation methods often give better results, because they have proved to be valuable and flexible computational tools to calculate the value of options with multiple sources of uncertainty or with complicated features. The main characteristic that makes simulation so attractive is its ability to cope with uncertainty in a very simple way. According to Cortazar (2000), the recent trend in modelling price uncertainty using multi-factor models is much easier to implement using standard simulation than using other numerical approaches. There are two most popular simulation methods: Monte Carlo simulation and Bootstrap experiment. The research will be based on Monte Carlo simulation.

Monte Carlo simulation is one of the most popular numerical method for pricing financial options and other derivative securities because of the availability of powerful workstations and recent advances in applying the tool (Charnes 2000; Tian *et al.* 2008). Moreover, Monte Carlo simulation is attractive relative to other numerical techniques because it is flexible, easy to implement and modify.

The aim of the article is to adapt Monte Carlo simulation method to predict prices of vanilla option contracts and compare them to real observed option prices and to prices calculated with an analytical Black – Scholes formula. The object of the research is pricing of option contacts.

Logical analysis and synthesis of scientific literature, comparative analysis and graphical modelling, simulation technique were used for the research.

2. Main concepts concerned with options

Different authors give the similar description of an option contract, all emphasizing the right to choose. An option can be described as an instrument giving its owner the right but not the obligation to buy or sell something at in advance fixed price. Options are available on a wide range of products, beginning from grain, raw materials and ending in financial assets, gold or real estate. In this article the main attention is paid on stock options.

There are two types of options – calls and puts. A call option gives the holder the right to buy specified quantity of the underlying asset at the strike price on or before expiration date. The writer of the option however, has the obligation to sell the underlying asset if the buyer of the call option decides to exercise his right to buy. A put option gives the holder the right to sell specified quantity of the underlying at the strike price on or before an expiry date (LIFFE 2004). The writer of a put option has the obligation to buy the agreed asset at the strike price if the buyer decides to exercise his right to sell. The option holder is the person who buys the right conveyed by the option. The option writer or seller is obliged to perform according to the terms of the option. Strike price or exercise price is the price at which the option holder has the right either to purchase or to sell the underlying asset. (Jarrow 1983)

There are three different terms for describing where an option is trading in relation to the price of the underlying asset. These terms are "at-the-money", "in-the-money", and "out-of-the money". At the money means that the current market value of the underlying asset is the same as the exercise price of the option. A call option is said to be in the money if the current market value of the underlying asset is above the exercise of the option. In the case of a put option current market value should be below the exercise price of the option. If the exercise price is above the current market value in the case of a call option and below in the case of a put option, the option is said to be out of the money. These options can be executed only at a lost. (Haugen 2001)

It is often useful to characterise an option in terms of its payoff to the purchaser of the option. The initial cost of the option is then not included in the calculation (Hull 2008).

If K is the strike price and S_T is the final price of the underlying asset, the payoff from a long position in a call option is

$$\max(S_T - K, 0) \tag{1}$$

This reflects the fact that the option will be exercised if $S_T > K$ and will not be exercised if $S_T \le K$. The payoff to the holder of a short position in the Call option is

$$\max(S_T - K, 0) = \min(K - S_T, 0) \tag{2}$$

The payoff to the holder of a long position in a Put option is

$$\max(K - S_T, 0) \quad (3)$$

And the payoff from a short position in a Put option is

$$\max(K - S_T, 0) = \min(S_T - K, 0) \quad (4)$$

The style of an option refers to when that option is exercisable. According to Options Clearing Corporation (OCC) there may be three different styles of options: American style, European style and capped options. An American style option may be exercised at any time prior to its expiration. European style option may be exercised only during a specified period before the option expires. Usually such an option is exercised on its expiration day. Capped options are not traded in every exchange. Their trading conditions are individually depending on the exchange they are traded. A capped option will be automatically exercised prior to expiration if the options market on which the option is trading determines that the value of the agreed asset at a specified time on a trading day reached the cap price of the option (Friedentag 2000).

3. Main principles of option pricing

Because of the complex valuation of option contracts the main scientific studies are devoted to analyse separate methods of options pricing (Hull 2008; Jarrow, Rudd 1983; Martin 2001). Depending on the requirements, the option pricing model can range in complexity from a simple binomial model, to Black-Scholes, to sophisticate analytical and simulation models.

The primary methods for pricing options are binomial trees and other lattice methods, such as trinomial trees, and finite difference methods to solve the associated boundary value partial differential equations. According to Jia (2009), due to the complexity of the underlying dynamics, analytical models for option pricing entail many restrictive assumptions, so for real-world applications approximate numerical methods are employed, these include the valuation of options, the estimation of their sensitivities, risk analysis, and stress testing of portfolios. But, in recent years the complexity of numerical computation in financial theory and practice has increased enormously, putting more demands on computational speed and efficiency.

The most popular valuation model for options is the Black-Scholes model. The model is based on the theory that markets are arbitrage free and assumes that the price of the underlying asset is characterized by a Geometric Brownian motion. This method is commonly used for pricing European options as there is an analytic solution for their price (Bampou 2008).

Another technique for pricing options is the binomial lattice model. In essence, it is a simplification of the Black-Scholes method as it considers the fluctuation of the price of the underlying asset in discrete time. This model is typically used to determine the price of European and American options (Bampou 2008).

Monte Carlo simulation is a numerical method for pricing options. It assumes that in order to value an option, we need to find the expected value of the price of the underlying asset on the expiration date. Since the price is a random variable, one possible way of finding its expected value is by simulation. This model can be adapted to price almost any type of option (Bampou 2008).

The main options pricing models contain five factors that are used to determine a theoretical value for an option and which have to be taken into account when pricing option contracts (Hull 2008):

1. market price of the underlying asset;
2. strike price;
3. time to expiration;
4. volatility of the underlying asset;
5. interest rates;
6. dividends expecting during the life of the option.

Market price and strike price. The payoff from a call option will be the amount by which the stock price in the market exceeds the strike price dealt with the option. Call options therefore become more valuable as the stock price increases and less valuable as the strike price increases. For a put option, the payoff on exercise is the amount by which the strike price exceeds the stock price (Laurence, Avellaneda 2000). So the put option becomes less valuable as the stock price increases and more valuable as the strike price increases.

Time to expiration. Both put and call American options become more valuable as the time to expiration increases. European put and call options do not necessarily become more valuable as the time to expiration increases. This is because it is not true that the owner of a long-life European option has all the exercise opportunities open to the owner of a short-life European option.

Volatility. The volatility of a stock price is a measure of how uncertain we are about future stock price movements. As volatility increases, the chance that the stock price will change in both directions increases. The value of both calls and puts therefore increase as volatility increases (Hull 2008; Martin 2001).

Risk-free interest rate. The risk-free interest rate affects the price of an option in a less clear-cut way. Without additional assumptions it is difficult to gauge the effect of increasing interest rates. Since increasing interest rates decrease the present value of the exercise price, there is a tendency for call values to increase and put values to decrease. It should be emphasized that these results assume that all variables remain fixed. In practice, when interest rates fall (rise), stock prices tend to rise (fall). The net effect of an interest rate change and the accompanying stock price change therefore may be different from that just given (Hull 2008; Jarrow, Rudd 1983).

Dividends. Dividends have the effect of reducing the stock price on the ex-dividend date. The values of call options are negatively related to the size of any anticipated dividend, and the value of a put option is positively related to the size of any anticipated dividend.

As vanilla options are traded in exchange markets, it is more possibilities to find historical information about real market prices. In the case of exotic options there is no much of such possibilities because these contracts in many cases are over the counter contracts. Choosing adequate for market conditions pricing model is crucially important.

In order to price option investor must distinguish between intrinsic value and time value of option contract. Intrinsic value is the value that any given option would have if it were exercised today. Basically, the intrinsic value is the amount by which the strike price of an option is in the money. It is the portion of an option's price that is not lost due to the passage of time (Wagner 2009). The following equations can be used to calculate the intrinsic value of a call or put option:

Call Intrinsic Value = Underlying Stock's Current Price – Call Strike Price (5)

Put Intrinsic Value = Put Strike Price – Underlying Stock's Current Price (6)

The intrinsic value of an option reflects the effective financial advantage that would result from the immediate exercise of that option. Basically, it is an option's minimum value. Options trading at the money or out of the money have no intrinsic value.

The second important driver is time value. Prior to expiration, any premium in excess of intrinsic value is called time value (What is an Option? 2012). Time value is also known as the amount an investor is willing to pay for an option above its intrinsic value, in the hope that at some time prior to expiration its value will increase because of a favourable change in the price of the underlying security (Wagner 2009). The longer the amount of time for market conditions to work to an investor's benefit, the greater the time value. The formula for calculating the time value of an option is:

Time Value = Option Price – Intrinsic Value (7)

Time value is basically the risk premium that the option seller requires to provide the option buyer the right to buy/sell the stock up to the date the option expires. It is like an insurance premium of the option; the higher the risk, the higher the cost to buy the option.

4. Monte Carlo simulation

Simulation methods can be very helpful when pricing options because prices of options do not have a simple closed form solution and efficient computational methods are needed to determine them. According to Gitman (2009), simulation is a statistics-based behavioural approach that applies predetermined probability distributions and random numbers to estimate risky outcomes. Another definition says that simulation is the imitation of a real world process of system. In finance, a basic model for the evolution of stock prices, interest rates, exchange rates, and other factors would be necessary to determine a fair price of a derivative security (Kaplan 2008). Simulations make assumptions about the behaviour of the system being modelled. Simulation is used because it transfers work to the computer.

Despite the fact that simulation methods are very useful, they might have some limitations too (Everaert 2011):

– Obtaining results may be computationally expensive

– The results may be imprecise
– The results are often hard to replicate
– The results are experiment – specific: as population must be specified, the results of a simulation cannot be generalised.

Monte Carlo simulation is one of the most popular numerical method for pricing financial options and other derivative securities because of the availability of powerful workstations and recent advances in applying the tool (Charnes 2000; Tian *et al.* 2008). Monte Carlo simulation is a flexible method whose applicability does not depend on the dimension of the problem and does not suffer from the curse of dimensionality (Ibanez and Zapatero 2004). As the Monte-Carlo method relies on the average result of thousands of independent stochastic paths, massive parallelism can be adopted to accelerate the computation (Tian *et al.* 2008).

Many papers use Monte Carlo simulation since the pioneering works of Boyle (1977), Tilley (1993) and Bossaerts (1989). All these methods price options with a finite number of exercise opportunities, as an approximation to true American options. In general, they try to approximate the value function, or the optimal exercise frontier, combining simulation and dynamic programming. Methods based on dimensionality reduction of the value function include Barraquand and Martineau (1995), Tilley (1993), Carr and Yang (1997), Raymar and Zwecher (1997) and others. Methods based on the parameterization of the optimal exercise frontier include, among others, Bossaerts (1989), Grant *et al.* (1997), Garcia (2003) and Andersen (2000). Approximation of the value function is used in Tsitsiklis and Van Roy (1999), Carriere (1996), Longstaff and Schwartz (2001), Longstaff *et al.* (2001), Haugh and Kogan (2001). Moreover, Broadie and Glasserman (1997a) use simulated trees, while Broadie and Glasserman (1997b) and Boyle *et al.* (2001) use a stochastic mesh method. A numerical comparison of different algorithms is presented in Fu *et al.* (2001).

In some important applications, Monte Carlo simulation is used to find an approximate solution to a complex financial problem, particularly European-style and exotic options for which no analytical pricing formula is available (DeFusco *et al.* 2001). A Monte Carlo method is a technique that involves using random numbers and probability to solve problems and simulates paths for asset prices (Kaplan 2008; Jia 2009). Monte Carlo simulation generates a sample by drawing from a hypothesised analytical distribution. One of the biggest advantages is that successive replications generate a collection of samples with the same distributional properties as the original data (Everaert 2011; Gitman 2009). Though, there are some disadvantages too, as results depend on whether the distributional assumption is correct, there is a slow rate of convergence, it is very time-consuming and computationally intensive.

Moreover, Monte Carlo simulation is attractive relative to other numerical techniques because it is flexible, easy to implement and modify, and the error convergence rate is independent of the dimension of the problem (Charnes 2000). Since the convergence rate of Monte Carlo methods is generally independent of the number of state variables, it is clear that they become viable as the underlying models (asset prices and volatilities, interest rates) and derivative contracts themselves (defined on path-dependent functions or multiple assets) become more complicated (Fu *et al.* 2001; Jia 2009). A key specification in Monte Carlo simulations is the probability distributions of the various sources of risk. The implications of different investment policy decisions can be assessed through simulated time. In addition, Monte Carlo simulation is widely used to develop estimates of Value at Risk (DeFusco *et al.*. 2001). This methodology simulates many times the profit and loss performance of the portfolio over a specified horizon.

Boyle (1977) was the first one who proposed a Monte Carlo simulation approach for European option valuation. The method is based on the idea that simulating price trajectories can approximate probability distributions of terminal asset values. Option cash flows are computed for each simulation run and then averaged. The discounted average cash flow using the risk free interest rate represents a point estimator of the option value.

There are several ways to increase estimation accuracy; the simplest one is to increment the number of simulating paths. However, efficiency may also be improved by using variance reduction techniques, including the control-variate and antithetic-variate approaches (Cortazar 2000; Bolia and Juneja 2005). It will be interested in this thesis only in increasing the number of simulating paths.

The main characteristic that makes simulation so attractive is its ability to cope with uncertainty in a very simple way. According to Cortazar (2000), the recent trend in modelling price uncertainty using multi-factor models is much easier to implement using standard simulation than using other numerical approaches.

5. Data and methodology

In order to adopt Monte Carlo simulation for option pricing Matlab software was used. The algorithms for simulation were based on the works of DeFusco *et al.* (2001), Everaert (2011), Zhang (2009), Goddard (2006a, 2006b). Options prices were found in 5 steps using Matlab software:
1) The characteristics of the option contract and underlying asset were specified. As the underlying asset the S&P 500 index was chosen.
2) The time grid was indicated. The horizon in terms of calendar time was taken and spited into a number of sub-periods. Calendar time divided by the number of sub-periods is the time increment, Δt.
3) Potential future asset paths were generated.
4) The payoff for each path for both European calls and puts were calculated.
5) In order to get the Option price discount back was made.

As the purpose was to compare prices acquainted with Monte Carlo simulation with the prices calculated using Black-Scholes model, the following formulas were used:

$$c = Se^{-q(T-t)} N(d_1) - Xe^{-r(T-t)} N(d_2), \tag{8}$$

$$p = Xe^{-r(T-t)} N(-d_2) - Se^{-q(T-t)} N(-d_1), \tag{9}$$

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \tag{10}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{11}$$

where c – premium of European call option;
p – premium of European put option;
S – stock price;
X – exercise price;
T-t – time to maturity;
r – risk free interest rate;
q – dividends;
σ – volatility of stock price;
N1, N2 – the cumulative normal distribution function.

The research was based on the analysis of S&P 500 index, since it could be said that it is the most liquid one in the market. Historical data used in this research cover the period of the year of 2011. To calculate the volatility, the VIX index was taken, which is very useful to calculate S&P 500 index option prices. Moreover, US 3 month T-Bills were chosen as a risk free rate. Finally, the real historical S&P 500 index options prices were taken in order to compare simulated prices.

6. Empirical results

S&P 500 index asset paths for three period groups were generated: 1) weekly, 2) monthly and 3) 50 days. Every time 10000 runs in Monte Carlo simulation were taken in order to get asset paths. As it was mentioned all period groups were taken from the year of 2011.

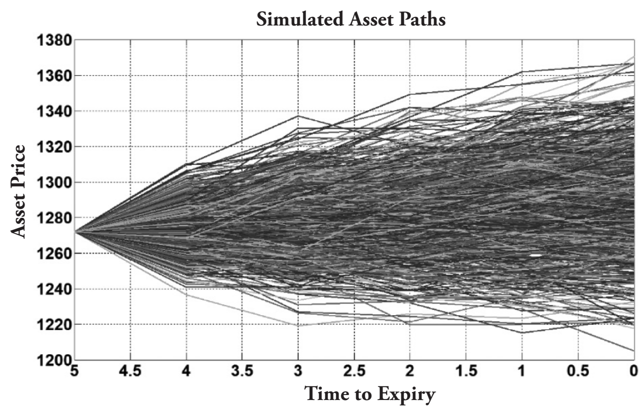


Fig.1. Asset path for a weekly (January 3-7) period

Source: Found by the authors

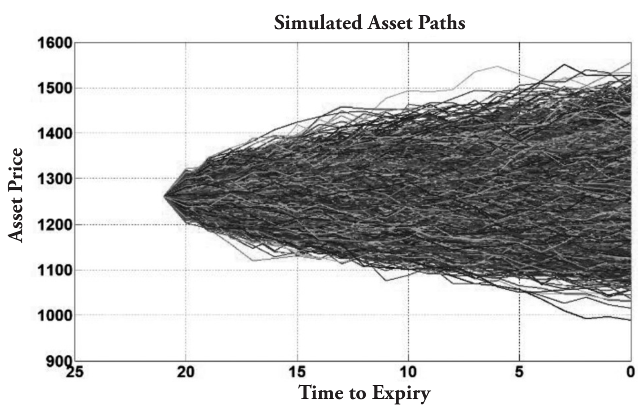


Fig.2. Asset path for a monthly (January) period

Source: Found by the authors

The first period group consists of 52 weeks or small periods. One week is considered to have 5 days, since only working days are important. The first weekly period is shown graphically in Figure 1. All other asset paths of weekly periods look very similarly.

The second period group consists of 12 months or little periods. One month 22 days on average, since weekends are excluded. Again, one of the monthly periods is shown graphically in Figure 2. All other asset paths for monthly periods have similar view.

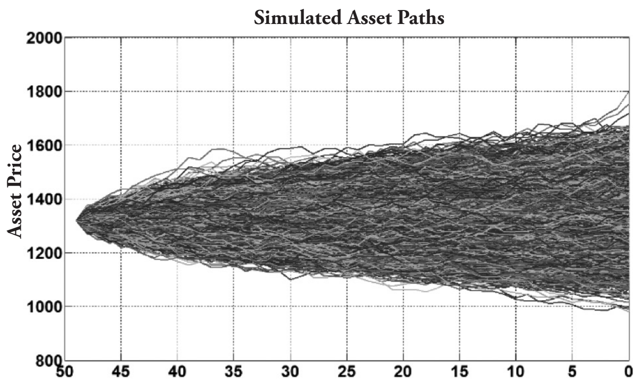


Fig.3. Asset path for a period of 50 days

Source: Found by the authors

The last group of 50 days periods consists of only six periods. Periods start at January 28, February 9, February 25, April 1, April 29 and May 11. Again every period asset path looks similar, so only one of these asset paths is also shown in the graph in Figure 3.

The prices of European Call and Put Options were calculated from equations no. 1 and 4 from this work. Prices were found for all period groups. The following tables show European call and put option prices generated from Monte Carlo simulation approach in very time period group.

Initially, it was started with the weekly period group. As it was mentioned before, weekends are excluded from every week, so there are only five days in a week. Since there are a lot of periods, there were created cycles for each 52 weeks in Matlab software in order to generate option prices.

Table 1. Simulated weekly European Call and Put prices

Weekly	European Call	European Put
07-Jan-11	8.39	10.12
14-Jan-11	26.08	1.54
21-Jan-11	39.84	0.46
28-Jan-11	12.53	6.65
04-Feb-11	28.49	1.48
11-Feb-11	4.89	13.84
18-Feb-11	18.12	3.23
25-Feb-11	18.19	6.41
04-Mar-11	26.75	2.58
11-Mar-11	61.49	0.13
18-Mar-11	8.99	18.93
25-Mar-11	30.64	1.56
01-Apr-11	13.63	6.53
08-Apr-11	15.16	4.82
15-Apr-11	35.67	0.52
22-Apr-11	1.05	27.82
29-Apr-11	48.58	0.07
06-May-11	9.80	8.75
13-May-11	9.43	8.37
20-May-11	43.31	0.24
27-May-11	6.57	12.08
03-Jun-11	40.42	0.42
10-Jun-11	30.71	1.44
17-Jun-11	7.99	15.72
24-Jun-11	45.45	0.44
01-Jul-11	0.03	66.32
08-Jul-11	87.34	0.00
15-Jul-11	1.42	33.44
22-Jul-11	44.33	0.42
29-Jul-11	22.43	5.41
05-Aug-11	20.15	9.98
12-Aug-11	48.58	6.72
19-Aug-11	30.91	11.02
26-Aug-11	15.05	26.29
02-Sep-11	46.13	3.90
09-Sep-11	10.97	30.88
16-Sep-11	22.50	15.40
23-Sep-11	28.90	12.66
30-Sep-11	30.24	15.25
07-Oct-11	50.11	6.27
14-Oct-11	22.67	11.92
21-Oct-11	15.88	19.62
28-Oct-11	25.09	8.10
04-Nov-11	35.98	6.05
11-Nov-11	8.17	29.35
18-Nov-11	30.69	8.98
25-Nov-11	36.08	6.91
02-Dec-11	13.34	18.22
09-Dec-11	4.33	36.59
16-Dec-11	84.32	0.10
23-Dec-11	10.47	13.60
30-Dec-11	36.43	2.02

Source: Found by the authors

Table 2 gives results for the simulated monthly prices of European options. Again, only working days are important, so one month may consist of 22 days on average.

Table 2. Simulated monthly Asian and European Call and Put prices

Monthly	European Call	European Put
Jan-11	7.90	43.99
Feb-11	11.66	38.21
Mar-11	19.97	32.36
Apr-11	13.91	28.87
May-11	43.41	8.30
Jun-11	6.64	57.26
Jul-11	138.67	0.30
Aug-11	55.11	29.61
Sep-11	52.64	32.55
Oct-11	8.44	95.73
Nov-11	38.86	36.13
Dec-11	31.35	26.72

Source: Found by the authors

The last table (Table 3) for European Options shows the simulated prices for periods of 50 days. There are only six periods.

Table 3. Simulated 50 days Asian and European Call and Put prices

Period starting at	Asian Call	Asian Put	European Call	European Put
28-Jan-11	13.11	29.9	32.08	46.2
9-Feb-11	19.93	16.96	39.93	32.27
25-Feb-11	11.37	29.2	28.89	43.8
1-Apr-11	40.35	4.28	59.04	14.91
29-Apr-11	22.04	15.28	40.85	29.81
11-May-11	24.31	13.41	43.41	28.02

Source: Found by the authors

Real historical option prices of S&P 500 index were taken in order to compare with European option prices simulated in Monte Carlo simulation. The historical prices were taken only for the year of 2011, because a longer time span would imply a tremendous amount of data. Moreover, data for 2011 is available only for the first 5 months, that is from the beginning of January, 2011 till the end of May, 2011. The data with observed prices contains S&P 500 index Call and Put options, with varying times to maturity and strike prices. Since the data consists of highest close bid and the lowest close ask, the average of both was used as an approximation of the price. The real observed data is grouped in the same time period groups as the simulated option prices.

Table 4 and the graph in the Figure 4 present weekly simulated and observed option prices. The real historical option prices were available only until the end of May, 2011, so the comparison is made only for the part of 2011. Moreover, some observed prices were missing during some weeks for the whole period.

Table 4. Simulated and Observed Option prices (weekly)

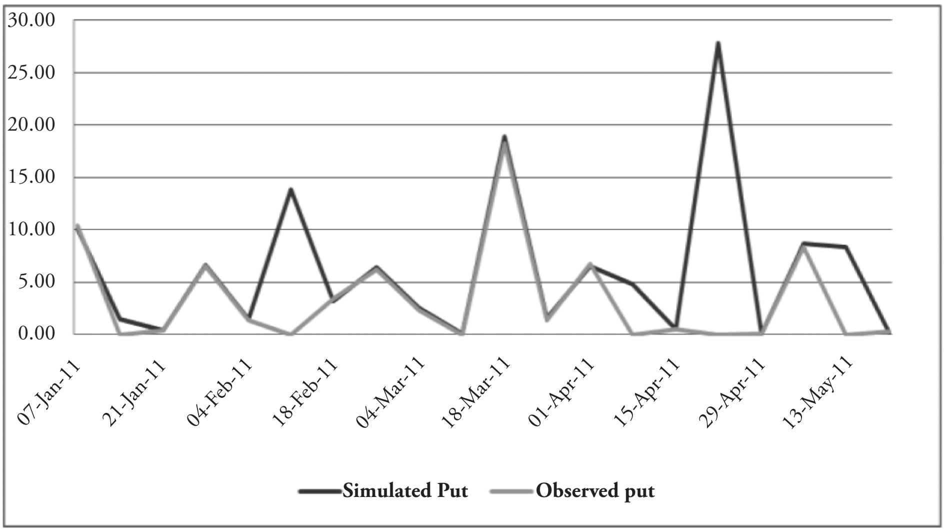
Weekly	Simulated		Observed	
	Call	Put	Call	Put
07-Jan-11	8.39	10.12	9.4	10.45
14-Jan-11	26.08	1.54	-	-
21-Jan-11	39.84	0.46	40.25	0.45
28-Jan-11	12.53	6.65	13	6.55
04-Feb-11	28.49	1.48	26.8	1.375
11-Feb-11	4.89	13.84	-	-
18-Feb-11	18.12	3.23	15.85	3.45
25-Feb-11	18.19	6.41	17.25	6.2
04-Mar-11	26.75	2.58	24.75	2.4
11-Mar-11	61.49	0.13	-	-
18-Mar-11	8.99	18.93	9.6	18.25
25-Mar-11	30.64	1.56	32.4	1.4
01-Apr-11	13.63	6.53	12.85	6.75
08-Apr-11	15.16	4.82	-	-
15-Apr-11	35.67	0.52	33.05	0.5
22-Apr-11	1.05	27.82	-	-
29-Apr-11	48.58	0.07	48.4	0.075
06-May-11	9.80	8.75	8.45	8.35
13-May-11	9.43	8.37	-	-
20-May-11	43.31	0.24	45	0.275

Source: Found by the authors

From the table and the graph it can be found that simulated call prices in most of the cases vary only in tenth and only some do not match at all. For example, there is a huge difference on March 11, when simulated call price is really high. However, this cannot be counted as deviation, since real observed data is not available on that date. The same happens in other dates, where observed data is not available, and it was considered as zero in the graph.

Talking about Put prices, it was found that simulated Put prices are higher than the observed ones during these periods: February 11 and April 22. Again, it happened because observed data is missing on those dates. Generally, it can be concluded that Monte Carlo simulation works pretty well when simulating option prices in weekly periods.

Table 5 and the graph in the Figure 5 give the results of monthly simulated and real historical option prices. Again, the observed option prices were available only until the end of May, 2011.



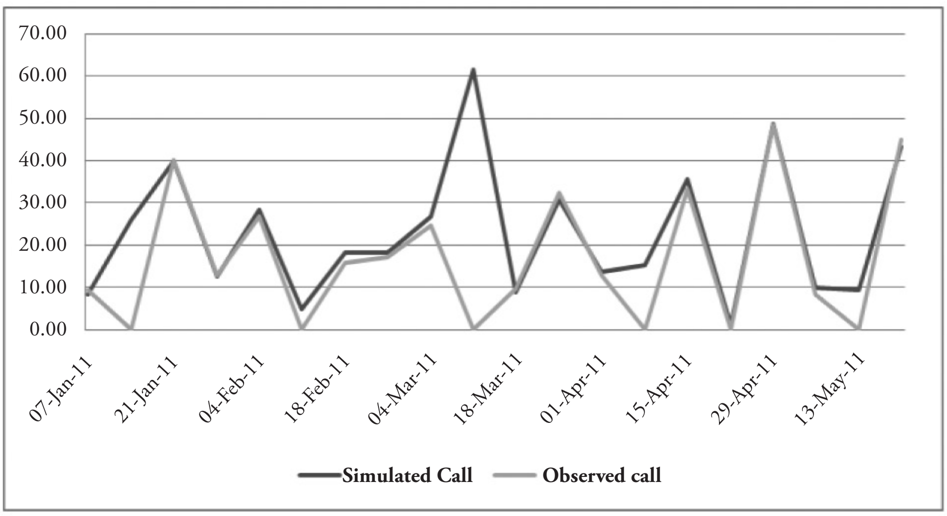


Fig.4. Simulated and Observed Option prices (weekly)

Source: Found by the authors

From the table and the graph it is possible to see that simulated and observed prices match almost perfectly, only decimal parts differ in most of the cases. To sum up, Monte Carlo simulation helps very well to predict Call and Put option prices in monthly periods.

When comparing simulated and real observed prices during the different periods of 50 days, very similar results like during monthly periods can be seen. Simulated and historical prices match almost very well. Most of the prices vary only in tenth except few. Moreover, simulated prices are higher than the real observed prices at the end of the graph. In summary, Monte Carlo simulation is quite accurate when predicting option prices for the longer periods.

Table 5. Simulated and Observed Option prices (monthly)

Monthly	Simulated		Observed	
	Call	Put	Call	Put
Jan-11	7.90	43.99	7.90	41.45
Feb-11	11.66	38.21	11.20	36.50
Mar-11	19.97	32.36	20.90	32.25
Apr-11	13.91	28.87	13.30	29.15
May-11	43.41	8.30	43.95	7.90

Source: Found by the authors

All in all, when comparing simulated option prices with real historical prices, the results were quite similar for the given period of time. They show that Monte Carlo simulation helps to predict options prices very well for either very short time periods or very long time periods too.

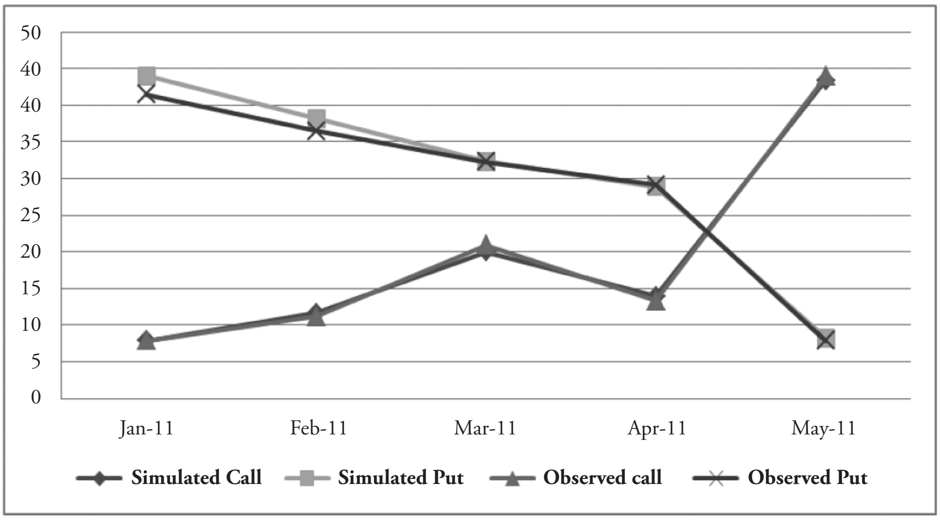
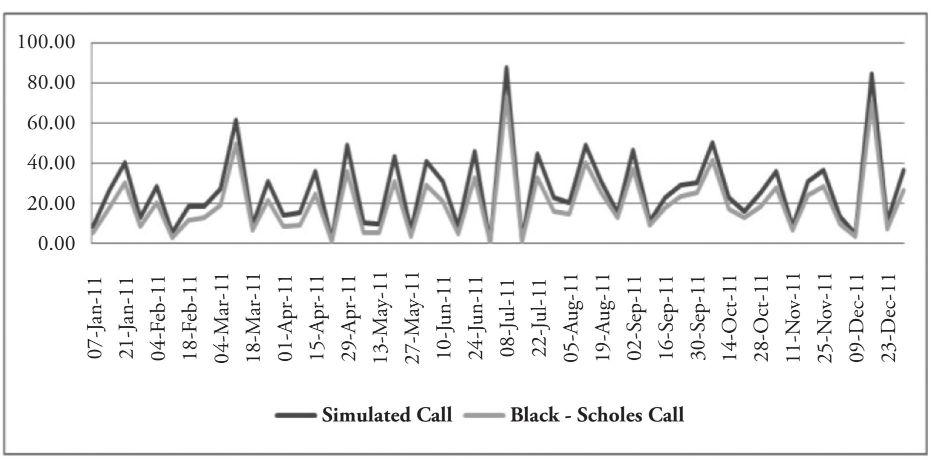


Fig.5. Simulated and Observed Option prices (monthly)

Source: Found by the authors

Using Black – Scholes equation the prices were calculated using real observed data of S&P 500 index from the year of 2011. The following tables and graphs give a comparison between simulated European option prices and calculated with Black – Scholes formula. The results from Black - Scholes are grouped in the same time period groups as the simulated option prices. Therefore, simulated prices and prices from analytical Black- Scholes formula should be compared in weekly and monthly time periods, as well as in 50 day time periods.



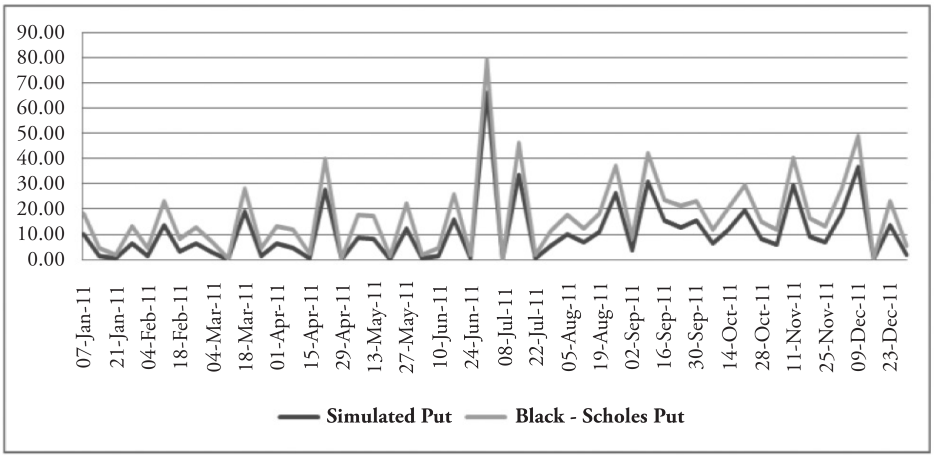


Fig.6. Weekly simulated and Black – Scholes option prices

Source: Found by the authors

Table 6. Weekly simulated and Black – Scholes option prices

Weekly	Simulated		Black - Scholes		Weekly	Simulated		Black - Scholes	
	Call	Put	Call	Put		Call	Put	Call	Put
07-Jan-11	8.39	10.12	5.06	18.22	**08-Jul-11**	87.34	0.00	73.07	0.01
14-Jan-11	26.08	1.54	17.85	4.65	**15-Jul-11**	1.42	33.44	0.78	46.32
21-Jan-11	39.84	0.46	29.93	1.85	**22-Jul-11**	44.33	0.42	32.56	2.05
28-Jan-11	12.53	6.65	8.07	13.12	**29-Jul-11**	22.43	5.41	15.40	11.55
04-Feb-11	28.49	1.48	19.95	4.41	**05-Aug-11**	20.15	9.98	14.66	17.65
11-Feb-11	4.89	13.84	2.58	22.99	**12-Aug-11**	48.58	6.72	40.31	12.47
18-Feb-11	18.12	3.23	11.34	8.28	**19-Aug-11**	30.91	11.02	24.93	18.39
25-Feb-11	18.19	6.41	12.40	12.78	**26-Aug-11**	15.05	26.29	12.38	37.01
04-Mar-11	26.75	2.58	18.63	6.73	**02-Sep-11**	46.13	3.90	36.72	8.59
11-Mar-11	61.49	0.13	49.26	0.75	**09-Sep-11**	10.97	30.88	8.89	42.14
18-Mar-11	8.99	18.93	6.43	28.01	**16-Sep-11**	22.50	15.40	17.82	23.64
25-Mar-11	30.64	1.56	21.56	4.76	**23-Sep-11**	28.90	12.66	23.23	21.32
01-Apr-11	13.63	6.53	8.51	13.37	**30-Sep-11**	30.24	15.25	25.03	23.34
08-Apr-11	15.16	4.82	8.94	11.73	**07-Oct-11**	50.11	6.27	41.28	12.00
15-Apr-11	35.67	0.52	24.39	2.47	**14-Oct-11**	22.67	11.92	16.70	20.43
22-Apr-11	1.05	27.82	0.46	39.95	**21-Oct-11**	15.88	19.62	12.74	29.46
29-Apr-11	48.58	0.07	35.72	0.63	**28-Oct-11**	25.09	8.10	18.51	15.13
06-May-11	9.80	8.75	5.30	17.72	**04-Nov-11**	35.98	6.05	27.74	12.00
13-May-11	9.43	8.37	4.98	17.21	**11-Nov-11**	8.17	29.35	6.23	40.38
20-May-11	43.31	0.24	30.96	1.56	**18-Nov-11**	30.69	8.98	23.59	16.23
27-May-11	6.57	12.08	3.49	22.15	**25-Nov-11**	36.08	6.91	28.48	13.19
03-Jun-11	40.42	0.42	28.94	2.11	**02-Dec-11**	13.34	18.22	9.52	27.99
10-Jun-11	30.71	1.44	20.89	4.75	**09-Dec-11**	4.33	36.59	3.11	49.11
17-Jun-11	7.99	15.72	4.74	26.06	**16-Dec-11**	84.32	0.10	69.79	0.56
24-Jun-11	45.45	0.44	32.85	2.09	**23-Dec-11**	10.47	13.60	6.67	23.24
01-Jul-11	0.03	66.32	0.01	79.04	**30-Dec-11**	36.43	2.02	26.47	5.64

Source: Found by the authors

Figure 6 and Table 6 show weekly simulated and Black – Scholes option prices. The results present that simulated Call option prices are a little bit higher than Call prices from analytical Black – Scholes formula whereas simulated Put option prices are a little bit lower. Generally, both simulated prices and prices calculated from analytical formula do not vary a lot as a result famous Black – Scholes formula can be replaced by Monte Carlo simulation when calculating option prices.

Table 7. Monthly simulated and Black – Scholes option prices

Monthly	Simulated		Black – Scholes	
	Call	Put	Call	Put
Jan-11	7.90	43.99	11.93	35.74
Feb-11	11.66	38.21	15.72	32.27
Mar-11	19.97	32.36	24.49	28.78
Apr-11	13.91	28.87	16.44	26.67
May-11	43.41	8.30	46.26	8.05
Jun-11	6.64	57.26	7.21	56.17
Jul-11	138.67	0.30	142.45	0.31
Aug-11	55.11	29.61	57.14	30.38
Sep-11	52.64	32.55	54.36	32.27
Oct-11	8.44	95.73	8.89	95.77
Nov-11	38.86	36.13	40.40	36.65
Dec-11	31.35	26.72	31.94	27.60

Source: Found by the authors

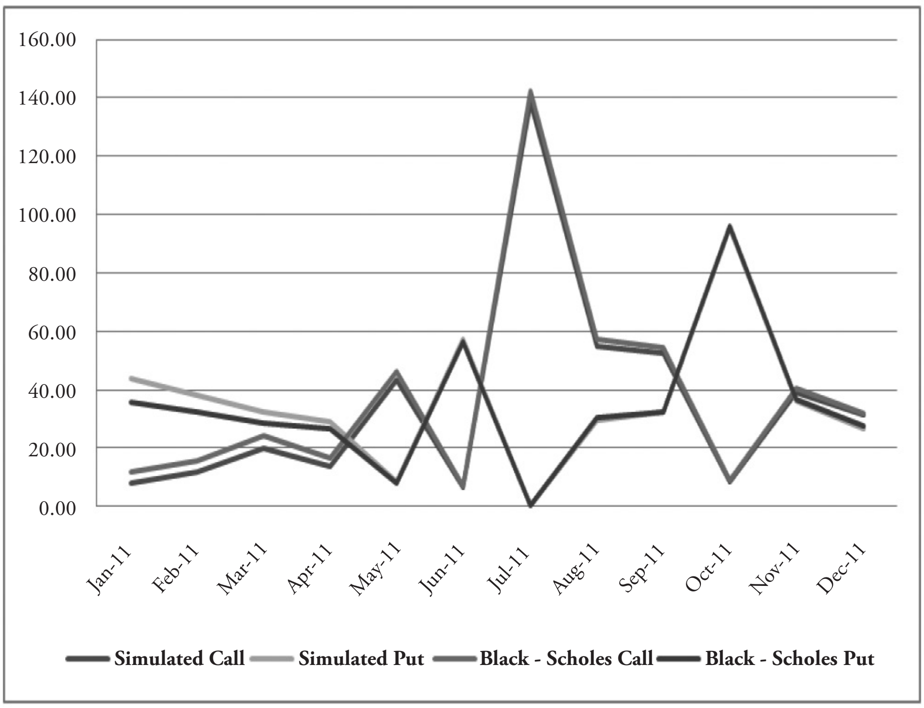


Fig.7. Monthly simulated and Black – Scholes option prices

Source: Found by the author s

Table 7 and Figure 7 present monthly simulated and Black – Scholes option prices. As it can be seen from the graph both simulated prices and prices calculated from analytical formula match together almost perfectly, only in the beginning of the year there are small deviations. Commonly, Monte Carlo simulation gives quite accurate results for option price prediction during the monthly periods.

Table 8. 50 days simulated and Black – Scholes option prices

Period starting at	Simulated		Black – Scholes	
	Call	Put	Call	Put
28-Jan-11	32.08	46.2	40.89	37.04
9-Feb-11	39.93	32.27	50.69	25.33
25-Feb-11	28.89	43.8	35.55	36.4
1-Apr-11	59.04	14.91	64.86	12.83
29-Apr-11	40.85	29.81	44.87	27.6
11-May-11	43.41	28.02	47.72	25.88

Source: Found by the authors

Finally, simulated and Black – Scholes option prices for the period of 50 days are given in Table 8 and Figure 8. Simulated Call prices and simulated Put prices move together with prices from analytical formula respectively. However, the graph shows that Black – Scholes Call option prices are higher than simulated ones, while Black – Scholes Put option prices are lower than simulated prices. This might tell that for longer periods Black – Scholes formula lead to some errors when the amount of data increases and Monte Carlo simulation becomes better method to calculate option prices. This also was discussed in literature in the papers of Jia (2009), Charnes (2000), Tian *et al.* (2008), Boyle (1977) and others.

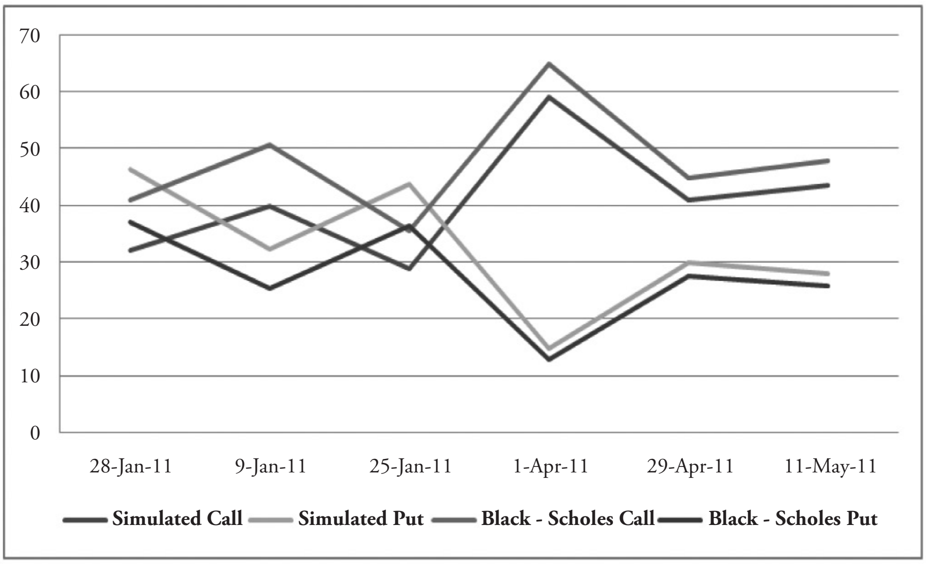


Fig.8. 50 days simulated and Black – Scholes option prices
Source: Found by the authors

To sum up, Monte Carlo simulation gives almost very good results when comparing with the results from analytical Black – Scholes formula. Both Call and Put option prices match very well especially in monthly periods. However, sometimes Monte Carlo simulation is even more accurate than the analytical model, especially in longer periods, like in this case of 50 days.

Conclusions

After the research of option pricing the following conclusions can be made:

The main distinguishing feature of the option contract is that his holder has possibility to choose if to use this contract. The writer of the option is in the opposite position; he has the obligation to fulfil the choice of the holder.

The main factors in option pricing are market price of the underlying asset, strike price, volatility of the asset, time to maturity of the contract, interest rates and dividends.

Option pricing is very essential area of research in financial community. Primary methods to calculate option prices remain binomial models and finite difference methods. So in most of the cases options can be valued analytically or using numerical combination. For example, the most known Black – Scholes model provides obvious closed form solutions for the values of certain call and put options.

In situations when numerical and analytical models become unavailable, simulation methods always give better results because they have proved to be valuable and flexible computational tools to calculate the value of options with multiple sources of uncertainty or with complicated features. The main characteristic that makes simulation so attractive is its ability to cope with uncertainty in a very simple way.

Monte Carlo simulation is one of the most popular numerical method for pricing financial options and other derivative securities because of the availability of powerful workstations and recent advances in applying the tool. Monte Carlo simulation proved to be very attractive technique, as it is flexible, easy to implement and modify.

The comparison between simulated European option prices and real observed option prices showed that simulated and historical prices in most of the cases matched almost perfectly and varied only in tenth. Generally, it can be concluded that Monte Carlo simulation helps pretty well when predicting option prices for either very short time periods or for longer time periods like 50 days.

The results presented that simulated Call option prices are a little bit higher than Call prices from analytical Black – Scholes formula whereas simulated Put option prices are a little bit lower in weekly periods. When discussing the outcomes during the monthly periods, there were found that simulated and Black –Scholes option prices match almost perfectly, only in the beginning of the graph there were small deviations. Commonly, Monte Carlo simulation gives quite accurate results for option price prediction during the short and medium periods.

During longer periods like 50 day periods, the results show that Black – Scholes Call option prices are higher than simulated ones, while Black – Scholes Put option prices are lower than simulated prices. This might tell that for longer periods Black – Scholes formula lead to some errors when the amount of data increases and Monte Carlo simulation becomes better method to calculate option prices.

Both simulated prices and prices calculated from analytical formula do not vary a lot and as a result famous Black – Scholes formula can be replaced by Monte Carlo simulation when calculating option prices. Monte Carlo simulation is sometimes even more accurate than the analytical model, especially in longer periods.

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